AD623047

Technical Research Note 159

AD

SOME TECHNICAL CONSIDERATIONS IN MULTIDIMENSIONAL SCALING

by ROBERT F. BOLDT

Statistical Research and Analysis Laboratory

AUGUST 1965

APRO

U. S. ARMY PERSONNEL RESEARCH OFFICE

U. S. ARMY PERSONNEL RESEARCH OFFICE

An activity of the Chief, Research and Development

J. E. UHLANER
Director of
Laboratories

M. O. BECKER
Colonel, GS
Commanding

NOTICES

AD

Technical Research Note 159

SOME TECHNICAL CONSIDERATIONS IN MULTIDIMENSIONAL SCALING

By Robert F. Boldt

STATISTICAL RESEARCH AND ANALYSIS LABORATORY
Cecil D. Johnson, Chief

U. S. ARMY PERSONNEL RESEARCH OFFICE

Office, Chief Research and Development
Department of the Army

Washington, D. C. 20315

August 1965

Army Project Number
2T013001A708

DRL 36

USAPRO Technical Research Reports and Technical Research Notes are intended for sponsors of R&D tasks and other research and military agencies. Any findings ready for implementation at the time of publication are presented in the latter part of the Brief. Upon completion of a major phase of the task, formal recommendations for official action normally are conveyed to appropriate military agencies by briefing or Disposition Form.

FOREWORD

A portion of the research effort of individual U. S. APRO research scientists is devoted to basic research designed to extend the capability of human factors research in terms of knowledge, techniques, and methodology which can ultimately be applied in seeking solutions to Army operational problems. This in-house laboratory independent research is conducted under the Department of Army Research and Development Project No. 2T013001A708, FY 1965 Work Program.

The present Technical Research Note reports on a study conducted within the Statistical Research and Analysis Laboratory. The study as originally conceived dealt with a methodology developed for multidimensional scaling in the definition and measurement of human values. While the scaling problem was not carried to completion because the data were found to be inadequate for the statistical model, the very difficulties encountered led to certain methodological advances which could facilitate further development in application of the method.

J. E. UHLANER
Director of Laboratories

SOME TECHNICAL CONSIDERATIONS IN MULTIDIMENSIONAL SCALING

BRIEF

In an examination of problems arising in multidimensional scaling, the multivariate analysis of interpoint distances into coordinates in m-dimensional Euclidean space is first described. Problems encountered in application of the method are then presented, and solutions developed or suggested by the writer are discussed.

The first difficulty was the occurrence of a complex additive constant during the course of Messick-Abelson interations for solutions of relatively low dimensionality. A regression approach is suggested as a reasonable alternative. This approach cannot yield a complex additive constant and has been successfully used where the more common method has failed.

A second problem arose because the common Hotelling method of extracting characteristic roots and vectors does not find the vectors in the algebraic order of the roots. This difficulty, combined with the relative slowness of the Hotelling method, recommends the use of Jacobi's method.

A third problem was the occurrence of negative "distances" obtained when the final additive constant is added to the scale values. It was conjectured that this result may arise in samples even when a proper dimensionality has been defined and the solution sought. A model sampling study of a one-dimensional system supported the conjecture. Hence, the occurrence of such negative "distances" is not necessarily indicative of an underlying structure of higher dimensionality.

Finally, it was pointed out that the occurrence of negative "distances" is actually built into the successive intervals and comparisons approaches. Alternative assumptions were discussed.

SOME TECHNICAL CONSIDERATIONS IN MULTIDIMENSIONAL SCALING

CONTENTS

TABLES

SOME TECHNICAL CONSIDERATIONS IN MULTIDIMENSIONAL SCALING

INTRODUCTION

Papers by Young and Householder (1) and Richardson (2) and later by Torgerson (3) and Messick and Abelson (4) mark the development of a methodology for representing the organization of persons' perceptions of classes of stimuli as a configuration of points in psychological space. The methods used rely on estimating as numbers the perceived differences among all pairs of a group of stimuli. Numbers representing differences of stimulus pairs are interpreted as distances among points. If the model is appropriate, the distances are considered Euclidean. Coordinates on a set of basic dimensions can then be found. These techniques have been used to analyze judgments on a wide variety of stimuli. Most such applications are not of an applied nature, although a more recent report by Siegel and Smith (5) describes an application of the method to a problem in job analysis. Because of the rapidly growing body of studies using the method and the anticipated expansion of practical applications, it is appropriate to describe certain problems that have been encountered in the use of the technique and to provide such solutions as have been obtained. Prior to this description it will be useful to describe the multivariate analysis utilized to translate interpoint distances into coordinates in m-dimensional Euclidean space.

DISTANCES INTO COORDINATES

As mentioned above, a key step of the multidimensional scaling procedure is to resolve interpoint distances into coordinates. If d_{ij} is the distance between stimuli i and j which are located in m-dimensional psychological space, then

$$d_{ij} = \sqrt{\sum_{k=1}^{m} (a_{ik} - a_{jk})^2} \tag{1}$$

where the a_{ik} is the coordinate of the ith point in the kth dimension. Squaring both sides of (1) and expanding the expression in parentheses on the right-hand side, obtain

$$d^2_{ij} = \sum_{k=1}^{m} a^2_{ik} + \sum_{k=1}^{m} a^2_{jk} - 2 \sum_{k=1}^{m} a_{ik}\, a_{jk}. \tag{2}$$

There would be $\frac{n(n-1)}{2}$ such equations where n is the number of stimuli.

The following matrices are defined:

D is an n x n matrix with elements d^2_{ij}.

L is an n x 1 column of 1's.

A is an n x 1 column with entries $\sum_{k=1}^{m} a^2_{ik}$.

$\tilde{F}$ is an n x m matrix of coordinates.

I is the n x n identity matrix.

$W = (I - (1/n)\, L\, L')$.

Then the equations (2) in matrix form are

$$D = A\,L' + L\,A' - 2\,\tilde{F}\,\tilde{F}'. \tag{3}$$

Premultiplying and postmultiplying both sides of (3) by W, noting that WL = 0, yields

$$W\,D\,W = -\,2\,W\,\tilde{F}\,\tilde{F}'\,W. \tag{4}$$

When the matrix operations involved in $W\tilde{F}$ are traced, the result is seen as the subtraction of a constant from each column of $\tilde{F}$, so that the sum of each column is zero (the constant will usually be different for each column). Geometrically, this subtraction has the effect of placing the origin of the space at the centroid of the points and has no effect on the interstimulus distances. We therefore define $F = W\tilde{F}$ and modify equation (4) to

$$F\,F' = -\,(1/2)\,W\,D\,W. \tag{4a}$$

Then, if D is known, a usual factorial decomposition of the right-hand side of (4a) will yield the coordinates within an arbitrary centering and orthogonal rotation. Unfortunately, with the exception of methods used by Helme (6), or empirical methods (7), the elements of D are not known because of the indeterminacy of an additive constant (4) which is due to the nature of the way in which scale values of differences are obtained (8). Both the method of successive intervals and the method of complete triads (3) yield interpoint distances only up to a linear transformation. Neglecting any multiplicative indeterminacy, if s_{ij}

is the scale value associated with the dissimilarity of the ith and jth stimuli, then the additive indeterminacy c can be expressed by relating the d's and s's as follows.

$$d_{ij} = s_{ij} + \delta_{ij}\, c. \tag{5}$$

Where $\delta_{ij} = 0,\ i = j$

and $\delta_{ij} = 1,\ i \neq j$

$s_{ij} = 0,\ i = j$

Squaring both sides of equation (5), one can obtain in matrix form

$$D = B + c\,E + c^2(L\,L' - I), \tag{6}$$

where B is an $n \times n$ matrix with entries equal to s_{ij}^2, and E is an $n \times n$ matrix with entries equal to s_{ij}. Substituting the right-hand side of (6) for D in equation (4a) yields

$$F\,F' = -(1/2)(WBW + cWEW - c^2 W). \tag{7}$$

The unknowns in equation (7) are the entries in F and the constant c. Computing experience indicates that the characteristic vectors of the right-hand side of (7) are relatively insensitive to the choice of value of c. Hence one could guess a value of c and then extract the first m factors of the matrix indicated by the right-hand side of (7). For example, suppose X_ℓ is the ℓth such characteristic vector and M is the matrix obtained from (7) using a particular value for c. Then $X_\ell' M X_\ell$ would yield the associated characteristic root. One could take advantage of the relative invariance of the characteristic vectors with change in c, once having found the first m X's for a particular value of c, by estimating the sum of characteristic roots as a quadratic function with c as an argument, i.e., if R(c) is the sum of characteristic roots, then

$$R(c) = \left(-\frac{1}{2}\right)\left[\sum_{k=1}^{m} X_k' WBW X_k + c \sum_{k=1}^{m} X_k' WEW X_k - mc^2\right]. \tag{8}$$

Similarly, if S(c) is the sum of the diagonals of the right-hand side of equation (7) with c as an argument, then

$$S(c) = -(1/2)((1/n^2)b_{..}-(1/n)b_{i.}-(1/n)b_{.j}+c((1/n^2)e_{..}-(1/n)e_{i.}-(1/n)e_{.j}) -(n-1)c^2). \tag{9}$$

Where b_{ii} and e_{ii} are the diagonals of B and E respectively. To find c, Messick and Abelson (4) equate R(c) and S(c) and solve the resulting quadratic equation for a new value of c which is then used to compute a new M which is used to get new X's, etc. This procedure, in effect, sets the sum of the n - m remaining roots of M equal to zero. If these roots are small, then the lower rank approximation of M by FF' will be good.

COMPLEX ADDITIVE CONSTANT

The Messick-Abelson additive constant procedure is an ingenious one by which many scaling problems have been solved. However, in several studies, the author has observed procedural difficulties which need to be resolved. These difficulties seem to arise when the chosen dimensionality is small relative to the number of stimuli used. To choose a dimensionality, the investigator guesses some value for c, computes the right-hand side of (7), factors the resulting matrix, and examines the profile of characteristic roots. Where there seems to be a sharp break in the size of the roots, he sets a trial dimensionality and begins additive constant iterations as described above. For example, Boldt (9) followed this procedure on a 15-stimulus problem and found one very large root, the remaining roots being relatively small. However, when single factor iterations were initiated, they could not be completed because of the complex roots of the quadratic equation obtained when R(c was equated to S(c). Since the analytical procedure as currently designed depends entirely on real numbers, there was no way to proceed. To solve this problem, additional dimensions were added, although the additional dimensions in no way added to the substantive understanding of the study. Complex constants were also encountered by Wiskoff (10) and Olans (11).

To bypass this problem, a procedure is needed for estimating the additive constant which always yields a real number. To find such a method, we first substitute s's and c for d in equation (1) obtaining

$$s_{ij} + c = \sqrt{\sum_{k=1}^{m}(a_{ik} - a_{jk})^2} \qquad i \neq j. \tag{10}$$

Guessing a value for c, the right-hand side of (7) can be constructed and factored to obtain F for the guessed c and dimensionality. If f_{ik}

is a typical entry in F, then $\sqrt{\sum_{k=1}^{m} (f_{ik} - f_{jk})^2}$ can be used as a trial estimate of the right-hand side of (10), with the following modification required. Knowing in advance that the constant c will be modified, we can allow for a change in the scale of the distances reconstructed after the next iteration by allowing the estimated distances to be expanded or contracted through multiplication by a constant, q. That is, write

$$s_{ij} + c = \tilde{q} \sqrt{\sum_{k=1}^{m} (f_{ik} - f_{jk})^2} \qquad i \neq j. \tag{11}$$

Since the s's are known and f's are determined using an old c, the constant of regression of s on $\sqrt{\Sigma(f_{ik} - f_{jk})^2}$ can be found and is equal to minus the new additive constant. This new constant could be used to get new f's and the procedure iterated until the constant ceases to change. When convergence is achieved, the coordinates, a's, are equal to b times the final F.

Clearly, no complex numbers can be involved in this process unless b is negative, i.e., the theoretical distances correlate negatively with the s's. If this happens, it seems reasonable to suspect that something is drastically wrong with the whole conceptualization, provided all computations are right. One way to correct this condition is to reverse the sign of the s's, although it is very difficult to imagine how such a change could be reasonable in the context of multidimensional scaling. No such negative correlation is known to the author.

CHOICE OF FACTORING METHOD

As indicated above, the multidimensional analysis requires guessing a constant, using it to compute the right-hand side of equation (7) and factoring the resulting matrix. Since the regression procedure does not involve calculating characteristic roots, perhaps centroid or square root factoring would be acceptable, although with computers the extraction of characteristic roots and vectors is quite feasible and in fact routine. If characteristic roots and vectors are used, there is some importance in choosing a proper factoring method. Since the number of factors desired is usually less than the order of the matrix to be factored, one would be tempted to use Hotelling iterations (12) which extract one factor at a time. However, this method extracts factors in the order of the absolute value of the characteristic roots. Messick and Abelson (4) have reported some results which render extraction in the order of the absolute value of the roots somewhat less desirable. They have pointed out that not only are the characteristic vectors from (7) relatively insensitive to changes in c, but also that the associated roots remain in the same order, although the magnitude and separation of roots may change as c changes.

Hence, if one makes a sufficiently bad initial guess at the value of c, the matrix to be factored may have strong negative roots. While this problem is not a serious one from the standpoint of numerical analysis, the practitioner should be aware that it can be encountered in practice. If Hotelling interations (12) are used, these large (in absolute value) negative roots will be extracted. Such roots are incompatible with the distance interpretation of the scale value. Hence the factoring method to be used should extract factors in the algebraic order of the characteristic roots rather than their absolute value. The Jacobi Method (13) is quite adequate in this regard, since it extracts all the factors which can then be selectively chosen in the algebraic order of the roots. This method has also proved to be considerably faster than the Hotelling method.

The difficulty in the order of extracting factors could probably be circumvented if one first did a complete factoring with some guessed value of the additive constant and visually inspected the resulting profile of characteristic roots. However, there is merit to computing routines such that one inputs the scale values, and the computer then works its way to solutions through increasing dimensionality until a satisfactory solution is achieved. The routine currently used by the author uses zero as a guessed constant no matter what the values of the s's and then uses as a starting constant for higher dimensions the final constant obtained from the solution at the next lowest dimensionality. The inclusion of the Jacobi routine with the factors taken in the algebraic order of the roots has been consistently successful.

To speed up the numerical procedure, an attempt was made to capitalize more fully on the relative invariance of the characteristic vectors with change in c. Suppose the matrices M and $\widetilde{M}$ are calculated as indicated on the right-hand side of equation (7) using two different values for c. Then

$$M = P\,G\,P'$$

and

$$\widetilde{M} = \widetilde{P}\,\widetilde{G}\,\widetilde{P}'$$

where P and $\widetilde{P}$ are the matrices of characteristic vectors and G and $\widetilde{G}$ are diagonal matrices of characteristic roots. The invariance which has been referred to is that

$$\widetilde{P} = P + E \tag{12}$$

where the entries in E are small in absolute value. Note that since $\widetilde{P}$ is orthogonal, the multiplication $\widetilde{P}'\,\widetilde{M}\,\widetilde{P}$ yields a diagonal matrix. It can be expected that the multiplication $P'\,\widetilde{M}\,P$ would nearly diagonalize $\widetilde{M}$ since

$$\begin{aligned} P'\,\widetilde{M}\,P &= (\widetilde{P} - E)'\,\widetilde{M}\,(\widetilde{P} - E) \\ &= \widetilde{P}'\,\widetilde{M}\,\widetilde{P} - \widetilde{P}'\,\widetilde{M}\,E' - E'\,\widetilde{M}\,\widetilde{P} + E'\,\widetilde{M}\,E \\ &= \widetilde{G} - (\text{terms in } E). \end{aligned}$$

Since $\tilde{G}$ is diagonal, and since E is small, it would be supposed that the diagonalization of $\tilde{M}$ would be nearly accomplished. Hence use of the old characteristic vectors could be expected to reduce time spent in diagonalizing new M matrices. This conjecture is recorded because at some time it may lead to useful application. However, for the problems on which it was attempted it was found that the Jacobi factoring from scratch was faster, since the required matrix multiplication $P' \tilde{M} P$ was very slow when done in FORTRAN with double subscripts. It is suggested that speed-up could occur if the user of multidimensional scaling had available faster techniques for accomplishing matrix multiplication.

NEGATIVE DISTANCES

Once a solution has been reached, there are two ways of estimating interstimulus distances. One of these is to use the Euclidean distance function (1) and the final coordinates; the other, more commonly used, is to add the constant **c** to the empirical scale values. The latter method has yielded negative numbers for solutions of relatively low dimensions in the studies of Boldt (9), Wiskoff (10), and Olans (11). Since negative numbers are incompatible with a distance interpretation, solutions at higher dimensions were sought. In the interest of parsimony, however, it is useful to inquire as to what extent these negative "distances" arise as a consequence of sampling fluctuation. To accomplish this inquiry, several successive interval multidimensional scaling samples were simulated using model sampling (14) procedures. Table 1 gives the matrix of interpoint distances used.

Table 1

INTERPOINT DISTANCES MATRIX

Stimulus	Stimulus					
	A	B	C	D	E	F
A	0	0	.5	1	2	4
B	0	0	.5	1	2	4
C	.5	.5	0	.5	1.5	3.5
D	1	1	.5	0	1	3
E	2	2	1.5	1	0	2
F	4	4	3.5	3	2	0

If stimulus A is located at the origin, and entries in the first row of Table 1 are taken as coordinates in a one-dimensional Euclidean space, these coordinates can generate the rest of the table. Hence a one-dimensional system is implied in Table 1. Consistent with the successive interval models (15), the distances were assumed to be normally distributed over presentations. Category boundaries were set at 2,4,6,8, and 10.

Then data for six samples of 20 subjects each were generated and scale values found assuming discriminal dispersions of 20. Multidimensional analysis of the resulting scale values was then accomplished using procedures described above. Only one of the samples gave all positive distances when the additive constant from a one-dimensional solution was added to the scale values. Four of the samples required two dimensions, and the fifth sample would have required five dimensions. The problem was, of course, rigged to produce the result--it was expected that the negative "distance" would occur at the zero distance between stimuli A and B. However, every stimulus pair yielded a negative distance for at least one sample. A majority of the pairs yielded negative "distances" in one or more of the cases where two dimensions were adequate. These results may well be peculiar to the particular configuration of parameters, especially including sample size. The implication is clear, however, that the occurrence of negative "distances" does not necessarily imply a higher dimensionality of the underlying stimulus configurations. Additional research, beyond the scope of the present study, is needed on which reasonable sets of parameters could be used to determine the extent to which these negative "distances" would be due to sampling fluctuation in practical experimentation.

DISTRIBUTION OF INTERPOINT DISTANCES

The results of the model sampling above deal with the case where scale values are obtained by successive intervals techniques and would probably apply to comparisons methods (3). They do not apply to graphic rating procedures. In a personal communication in 1956, Tucker pointed out that both the successive intervals and comparisons methods, as used in practice, assume that stimulus strengths are normally distributed over presentations. In the multidimensional scaling problem, the stimulus strengths are interpreted as interpoint distances. Since the normal distribution has non-zero probability ordinates over the entire real axis, it follows that the occurrence of negative distances is implicit in both the successive intervals and comparisons models. It should not be surprising that negative "distances" occur in some solutions. Assumptions other than normality could be tried. For example, the author (16) has used gamma distribution assumptions to obtain estimates of the distances on a ra io scale, but in that attempt there was no good rationale for the assumption, and the occurrence of complex solutions severely limited the value of the approach. Tucker has pointed out that if, over a large number of presentations, the stimulus points are normally distributed in m-dimensional psychological space, the interpoint distances would take on a non-central chi-square distribution. However, no feasible numerical techniques based on the non-central chi-square are currently available.

LITERATURE CITED

1. Young, G. and Householder, A. S. Discussion of a set of points in terms of their mutual distances. Psychometrika, 3, 1938. pp 19-22.

2. Richardson, M. W. Multidimensional psychophysics. Psychological Bulletin 35, 1938. pp 659-660.

3. Torgerson, W. S. Multidimensional scaling: I. Theory and method. Psychometrika, 17, 1952. pp 401-419.

4. Messick, S. J. and Abelson, R. P. The additive constant problem in multidimensional scaling. Psychometrika, 21, 1956. pp 1-17.

5. Siegel, A. I. and Smith, R. A multidimensional scaling analysis of the job of civil defense director. Prepared for Office of Civil Defense, Department of the Army, Contract OCD-PS-64-30. 1965.

6. Helm, C. E. A multidimensional rating scaling analysis of color relations. Educational Testing Service Technical Report. 1959. Princeton Univ., Princeton, N. J.

7. Mellinger, J. An investigation of psychological color space. Unpublished doctoral dissertation. University of Chicago. 1956.

8. Messick, S. J. An empirical evaluation of multidimensional successive intervals. Psychometrika, 21, 1956. pp 367-375.

9. Boldt, R. F. Construction of some judgment spaces. Research Bulletin 61-18. December 1961. Educational Testing Service. Princeton, N. J.

10. Wiskoff, M. S. A multidimensional representation of psychologists' perceptions of psychologists. Unpublished doctoral dissertation. Maryland University. 1963.

11. Olans, J. L. Dimensionality of selected TAT cards. Unpublished doctoral dissertation. George Washington University. 1965.

12. Hotelling, H. Analysis of a complex of statistical variables into principal components. Journal of Educational Psychology, 24, 1933. pp 417-441.

13. Green, B. F. Digital Computers in Research. New York: McGraw-Hill. 1963. pp 150-152.

14. Meyer, H. A. (Ed.) Symposium on Monte Carlo Methods. New York: John Wiley and Sons, Inc., 1956. pp vii-viii.

15. Torgerson, W. S. <u>Theory and Methods of Scaling</u>. New York: Wiley and Sons, 1958.

16. Boldt, R. F. A ratio scaling model. <u>American Psychologist</u>, <u>12</u>, 1957. p 469.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY *(Corporate author)* U. S. Army Personnel Research Office	2a REPORT SECURITY CLASSIFICATION Unclassified 2b GROUP

3 REPORT TITLE

Some Technical Considerations in Multidimensional Scaling

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5 AUTHOR(S) *(Last name, first name, initial)*

Boldt, Robert F.

6 REPORT DATE	7a TOTAL NO OF PAGES	7b NO OF REFS
August 1965	17	16

8a CONTRACT OR GRANT NO b. PROJECT NO DA R&D PJ 2T013001A708 c DRL d 36	9a ORIGINATOR'S REPORT NUMBER(S) Technical Research Note 159 9b OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10 AVAILABILITY/LIMITATION NOTICES

Qualified requestors may obtain copies of this report directly from DDC. Available, for sale to the public, from the Clearinghouse for Federal Scientific and Technical Information, Department of Commerce, Springfield, Virginia 22151.

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
	OCRD

13 ABSTRACT

Three problems encountered in executing the analysis of multidimensional scaling data are presented and their resolutions discussed. The first problem is that complex solutions to a quadratic equation may be encountered during Messick-Abelson additive constant iterations. An alternative iteration sequence is presented which necessarily involves only real solutions. This sequence has been successfully used repeatedly. The second problem is that with a poor initial guess at the additive constant, large negative characteristic roots may occur in matrices which theoretically represent scalar products of coordinates of points located in Euclidean space. A factoring method which simply omits these roots will yield a satisfactory solution. The third problem is that when the final additive constant is added to empirically obtained interpoint distances on a ratio scale, the resulting "distance" is often negative. A model sampling study using successive intervals assumptions demonstrates that such negative "distances" can arise as a consequence of sampling fluctuations, and hence, that the occurrence of such distances in not necessarily indicative of an underestimate of the dimensionality of the configuration of stimulus points.

DD FORM 1 JAN 64 1473

Unclassified

Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
*Scaling						
*Multidimensional scaling						
Multivariate analysis						
Statistics						
*Psychometrics						
Factor analysis						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

10. **AVAILABILITY LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.